

Real-Time Flow Forecasting

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The Chapter discusses the modelling of rainfall-flow (rainfall-runoff) and flow routing processes in river systems within the context of real-time flood forecasting. It is argued that deterministic, reductionist (or ‘bottom-up’) models are inappropriate for real-time forecasting because of the inherent uncertainty that characterizes river catchment dynamics and the problems of model over-parametrization. The advantages of alternative, efficiently parameterized *Data-Based Mechanistic* (DBM) models, identified and estimated using statistical methods, are discussed. It is shown that such models are in an ideal form for incorporation in a real-time, adaptive data assimilation and forecasting system based on recursive state space estimation (an adaptive version of the stochastic *Kalman Filter* algorithm). An illustrative example, based on the analysis of daily data from the ephemeral Canning River in SW Australia, demonstrates the utility of this methodology and illustrates the advantages of incorporating real-time state and parameter adaption.

Keywords: rainfall-flow processes, data-based mechanistic models, recursive estimation, Kalman filter, real-time forecasting, parameter adaptation, heteroscedasticity, variance adaptation

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1. Introduction

The primary objective of this Chapter is to describe recent research on the design of real-time, adaptive forecasting procedures for the prediction of flow (discharge) or river level (stage) in river systems. In particular, the aim is to produce an on-line, real-time approach to flow forecasting that is inherently stochastic and so able to predict not only the likely level of future flow, but also the uncertainty associated with this prediction. In this manner, the probability of a flood occurring in the near future is quantified and this additional information can then be used as a basis for decision-making, operational management and risk assessment in relation to the flooding of flood-prone locations.

The methodology described in subsequent sections of the Chapter can be applied to the forecasting of either river discharge or stage. For simplicity, however, ‘flow’ will be used here as a generic term to mean either of these two measures. Also, ‘flow’ will be taken to mean the total flow in the river, not just the ‘run-off’ component of total flow. In this context, the approach to forecasting described here is model-based: i.e. depending on the nature of the catchment and the forecasting objectives, the flow forecasts are based on an appropriate combination of stochastic, dynamic models for the relationships between: (i) rainfall and flow; and (ii) flow at various locations along the river: i.e. ‘flow routing’ models. Both types of model are estimated statistically on the basis of the available rainfall-flow data using the inductive *Data-Based Mechanistic* (DBM) approach to stochastic modelling (see e.g. Young and Lees, 1993; Young and Beven, 1994; Young, 1998b & the prior references therein), where both the model structure and the associated parameters are inferred from the data with the minimum of prior assumptions.

It is important to stress that, despite this reliance on statistical inference, the DBM representation of hydrological time series is not an exercise in ‘black-box’ modelling. Indeed, unlike many statistically-based models (or alternative neural network and neuro-fuzzy black-box models), the DBM model is only considered acceptable if it has an internal structure that can be interpreted satisfactorily in a physically meaningful manner. On the other hand, the DBM models often exploit ‘systems’ elements, such as parametrically efficient (parsimonious) transfer function relationships that are then decomposed in a physically interpretable fashion in order to satisfy the requirements of DBM modelling. As such, DBM models are normally in an ideal form for conversion to a stochastic state space form and can be embedded easily within a state estimation algorithm, such as the *Kalman Filter* (Kalman, 1960), that then provides the main engine for state updating, data assimilation and real-time forecasting.

This statistical approach to model-based forecasting has the virtue of being inherently stochastic and, because it is formulated in Bayesian, recursive estimation terms (see e.g. Bryson and Ho, 1969; Young, 1984), it provides an ideal basis for real-time implementation and the introduction of *adaptive* procedures. Such adaptation is motivated by a view that the rainfall-flow and flow routing processes are inherently ‘nonstationary’: i.e. no completely fixed model with constant parameters will be able to characterize the catchment behaviour for all times into the future. As a result, it is argued that the forecasting system should be based on models that are able to adjust to any, normally small, changes in the catchment behaviour not predicted accurately enough by the initially estimated model.

The Chapter has another, underlying objective that is of deeper philosophical and methodological significance and is, in part, a response to the recent increased interest in the so-called ‘top-down’ (or ‘holistic’) approach to modelling hydrological systems (e.g. Jothityangkoon *et al.*, 2001, which follows from the earlier contributions of Klemes, 1983; Young, 2003. See also the special issue of *Hydrological Processes* devoted to this topic: Vol 17, No. 11, 2003). Interest in top-down modelling has been revived largely because the alternative ‘bottom-up’ or ‘reductionist’ philosophy that dominated much research during the last century, has failed to solve the many problems of modelling natural environmental systems, including hydrological processes. Top-down modelling in hydrology has its parallels in the environmental (e.g. Young, 1978, 1983; Beck, 1983) and ecosystems (e.g. Silvert, 1993 & the prior references therein) literature of the 1970s and early 1980s.

As far as the author is aware, these latter contributions were the first to emphasise the inherent dangers of ‘deterministic reductionism’; i.e the widely held view that a physically-based simulation model can be constructed on the basis of purely deterministic equations that reflect the modeller’s perception of the physical system and that this model can be validated satisfactorily against the available data. They also presented initial thoughts on a more objective, statistical approach to modelling stochastic systems that tries to avoid such dangers as much as possible. This, in turn, led eventually to the DBM approach which is much in sympathy with the tenets of deterministic top-down modelling (e.g. Jothityangkoon *et al.*, 2001) but is, of course, rather different in its methodological basis. The papers also adumbrated very similar anti-reductionist arguments that have appeared recently in the hydrological literature and express some of these same views within a hydrological context (Jakeman & Hornberger, 1993, Beven, 2000). Quite similar anti-reductionist views are also appearing in other areas of science: for instance, in a recent lecture (Lawton, 2001), the current chief executive of the *Natural Environment Research Council* (NERC) recounted the virtues of the top-down approach to modelling ecological systems (although, for some reason, he did not appear to accept that such reasoning could also be applied to other natural systems, such as the physical environment).

The top-down approach to modelling can be justified by the results of *Dominant Mode Analysis* (DMA: Young *et al.*, 1996; Young, 1999a). This shows that complex systems, including large, deterministic simulation models, can be emulated, often to a remarkable degree, by much simpler models that reflect the ‘dominant modal dynamics’ of the complex system or model, as shown in the practical examples described in these references. Given that the DBM model is based on response data from the hydrological system, it is not surprising that it reflects the dominant modal behaviour of the system. Indeed, it is interesting to note that DMA exploits the same identification and estimation algorithms used in DBM modelling, as described in subsequent sections of this Chapter.

Bearing the above comments in mind, a wider aim of the present Chapter is to promulgate the philosophy of ‘inductive’, dominant mode, DBM modelling as an alternative to the ‘hypothetico-deductive’ (and often reductionist) approach that has dominated much hydrological modelling research over the last century (see Young, 2002). Other recent publications that have concentrated more centrally on this topic within this wider modelling context and can be considered as adjuncts to the present paper are: Young (1998a,b; 1999a); Young & Pedregal (1998,1999a);

Young & Parkinson (2002); Young *et al.*(1996); Shackley *et al.*(1998); Parkinson & Young, (1998); Young *et al.*, 2004.

Finally, it is important to stress that the present chapter concentrates on the problem of modelling rainfall-flow processes for the purpose of real-time flow forecasting. The aim is not to produce a detailed stochastic model that is intended for ‘what-if’ studies and planning purposes as, for example, the models considered in Beven and Binley (1992), Romanowicz and Beven (1998) and Beven *et al.*(2000). The DBM model is, however, a special form of *Hybrid Metric-Conceptual* (HMC) model (see the next section and Wheater, 1993) and, if required, it can be synthesized in a form which allows for these alternative applications (e.g. Young, 2003). It is then closely related to other HMC models, such as the Bedford-Ouse (Young, 1974; Whitehead and Young, 1975) and IHACRES (Jakeman *et al.*, 1990) models. Indeed, because of its more objective, statistical approach to model structure identification, DBM modelling can help to justify and improve the more conceptual elements in such models.

2. The Categorization of Rainfall-Flow Models

Before considering DBM modelling in detail, it is useful to set the scene by considering the various approaches that have been used to characterize the nonlinear dynamic relationship between rainfall and river flow. This is one of the most interesting modelling problems in hydrology. It has received considerable attention over the past thirty years, with mathematical and computer-based models ranging from simple black-box representations to complex, physically-based catchment models. It would be impossible to review this enormous literature here. Fortunately, however, there are many books available that deal in whole, or in part, with this challenging area of science and engineering. Useful texts of this type are: Anderson & Burt (1985); Shaw (1994); Singh (1995); and Beven (2001). The latter book, in particular, provides a clearly written review of the whole topic that not only deals critically with many recent developments but also provides an excellent introduction to the subject at the start of the twenty-first Century.

Wheater *et al.* (1993) have categorized rainfall-flow models into the following four, broad types.

- *Metric Models*, which are based primarily on observational data and seek to characterize the flow response largely on the basis of these data, using some form of statistical estimation or optimization (e.g. Wood & O’Connell, 1985; Young and Wallis, 1985; Young, 1986). These include purely black-box, time-series models, such as discrete and continuous-time transfer functions, neural network and neuro-fuzzy representations (e.g Tokar & Johnson, 1999; Jang *et al.*, 1997). Often, such models derive from, or can be related to, the earlier unit hydrograph theory but this is not always recognized overtly.
- *Conceptual Models*, which vary considerably in complexity but are normally based on the representation of internal storages, as in the original Stanford Watershed Model of the nineteen sixties (Crawford & Linsley, 1966). However, assumptions about catchment-scale response are not often included explicitly, notable exceptions being TOPMODEL (Beven & Kirkby, 1979) and the ARNO model (Todini, 1996). The essential feature of all these models,

however, is that the model structure is specified *a priori*, based on the hydrologist/modeller's perception of the relative importance of the component processes at work in the catchment; and then an attempt is made to optimize the model parameters in some manner by calibration against the available rainfall and flow data.

- *Physics-Based Models*, in which the component processes within the models are represented in a more classical, mathematical-physics form, based on continuum mechanics solved in an approximate manner via finite difference or finite element spatio-temporal discretization methods. A well known example is the *Système Hydrologique Européen* (SHE) model (e.g. Abbot *et al.*, 1986). The main problems with such models, which they share to some degree with the larger conceptual models, are two-fold: first, the inability to measure soil physical properties at the scale of the discretization unit, particularly in relation to sub-surface processes; and second, their complexity and consequent high dimensional parametrization. This latter problem makes objective optimization and calibration virtually impossible, since the model is normally so over-parameterized that the parameter values cannot be uniquely identified and estimated against the available data (see below).
- *Hybrid Metric-Conceptual (HMC) Models*, in which (normally quite simple) conceptual models are identified and estimated against the available data to test hypotheses about the structure of catchment-scale hydrological storages and processes. In a very real sense, these models are an attempt to combine the ability of metric models to efficiently characterize the observational data in statistical terms (the 'principle of parsimony': Box & Jenkins, 1970; or the 'Occam's Razor' of antiquity), with the advantages of conceptual models that have a prescribed physical interpretation within the current scientific paradigm.

The models in the two middle categories, above, are often characterized by a large number of unknown parameters that need to be estimated ('optimized' or 'calibrated') in some manner against the observational rainfall-flow time series. Because the number of parameters is normally very large in relation to the information content of the data, however, such models are often over-parameterized and not normally identifiable, in the sense that it is impossible to estimate their parameters uniquely without imposing prior restrictions on a large subset of the parameter values prior to estimation (see e.g. Young *et al.*, 1996). The author and his co-workers have addressed these problems of over-parameterization and poor identifiability associated with large environmental models many times over the past quarter century (see previous references). And recently, Beven and his co-workers (e.g. Franks *et al.*, 1997) have revisited this idea within the hydrological context, using the term 'equifinality' rather than 'non-identifiability' to describe the consequences of such over-parametrization: namely the existence of many different parametrizations and model structures that are all able to explain the observed data equally well, so that no unique representation of the data can be obtained within the prescribed model set.

There appear to be two main reasons for these identifiability problems. First, any limitations of the observational data can be important, since the available time

series may not be sufficiently informative to allow for the estimation of a uniquely identifiable model form. In particular, the inputs to a system may not be ‘sufficiently exciting’ (see e.g. Young, 1984), in the sense that they do not perturb the system sufficiently to allow for unambiguous estimation of all the model parameters within an otherwise identifiable model structure. Secondly, even if the input does sufficiently excite the system, there are usually only a limited number of dynamic modes - the *dominant modes* of the system - that are excited to any significant extent; and the observed output of the system is dominated by their cumulative effect.

The importance of this dominant mode concept in model identification and estimation is illustrated by Appendix 1 of Young (2001b), which shows how the response of a 26th order hydrological simulation model can be duplicated with exceptional accuracy (0.001% error by variance) by a much simpler 7th order dominant mode model. This is typical of most high order linear systems and appears to carry over to nonlinear systems. For example, Young *et al.*(1996), Young (1998b) and Young and Parkinson (2002) have used similar analysis to show how the response of high order, nonlinear global carbon cycle simulation models are accurately reproduced by differential equation models of much reduced order. This is also reflected in other recent work on the simplification of global climate models (Hasselmann *et al.*, 1997; Hasselmann, 1998).

As a result of dominant modal behaviour, the identifiable order is normally quite low for hydrological systems, and many previous rainfall-runoff modelling studies (e.g. Kirkby, 1976; Hornberger *et al.*, 1985; Jakeman & Hornberger, 1993; Young, 1993, 1998b; Young & Beven, 1994; Young *et al.*, 1997a,b; Ye *et al.*, 1998) suggest that a typical set of rainfall-runoff observations contain only sufficient information to estimate up to a maximum of six parameters within simple, nonlinear dynamic models of dynamic order three or less. In the hydrological examples discussed later, for instance, there is clear evidence in the data of only two significant dominant modes between the effective rainfall input and the flow response (as described by a second order transfer function model with only four or five parameters): a ‘quick’ mode with a residence time (time constant) of a few hours; and a ‘slow’ mode, with a residence time of many hours.

By their very nature, both the metric and HMC approaches avoid many of these ‘large model’ problems. As a result, they provide a potentially attractive vehicle for real-time flood forecasting: they can be justified well in statistical terms and they are inherently simple in both structure and application. Such simplicity means that the forecasting system can be more easily optimized on a regular basis in order to ensure near-optimal performance. And, as we see later, it facilitates the incorporation of advanced features such as on-line state and parameter adaptation. Of the two approaches, however, the attractiveness and practical utility of the basic metric model as a vehicle for flood forecasting is marred by its lack of any clearly defined internal physical interpretation.

For instance, neural network (e.g. Tokar & Johnson,1999) and neuro-fuzzy models have attracted a great deal of attention in recent years but they are the epitome of black box modelling, revealing very little of their internal structure that has any physical meaning (see the discussion in Young (2001c) on the paper by Hu *et al.*(2001), where a neuro-fuzzy model with 102 parameters can be replaced by a nonlinear DBM model with only 15 parameters if the internal structure of the

model is identified and taken into consideration). Neural models can also be misleading since their efficacy tends to be judged on the basis of their ‘one-step-ahead’ prediction performance, which often obscures underlying limitations in the full ‘simulation’ behaviour of the model. Indeed, in most cases, neural models are only predictive devices and cannot be used in a simulation mode. For these and other reasons, many hydrologists tend to mistrust such black box models as a basis for something as important as flood forecasting. Moreover, their lack of any obvious internal physical meaning means that metric models are difficult to interrogate and diagnose when errors are encountered. HMC models, on the other hand, do not suffer from these problems and, indeed, are often simpler in dynamic terms than the metric model.

Within the category of HMC models two main approaches to modelling can be discerned; approaches which, not surprisingly, can be related to the more general deductive and inductive approaches to scientific inference that have been identified by philosophers of science from Francis Bacon (1620), to Karl Popper (1959) and Thomas Kuhn (1962). In the first ‘hypothetico-deductive’ approach, the *a priori* conceptual model structure is effectively a theory of hydrological behaviour based on the perception of the hydrologist/modeller and is strongly conditioned by assumptions that derive from current hydrological paradigms (e.g. the Bedford Ouse and IHACRES models mentioned previously). The alternative DBM approach is basically ‘inductive’, in the sense that it tries to avoid theoretical preconceptions as much as possible in the initial stages of the analysis. In particular, the model structure is not pre-specified by the modeller but, wherever possible, it is inferred directly from the observational data in relation to a more general class of models (normally ordinary differential equations of the discrete-time equivalents). Only then is the model interpreted in a physically meaningful manner, most often (but not always) within the context of the current hydrological paradigm: e.g. the models of rainfall-flow data in Young (1993, 1998b), Young & Beven (1994) and Young *et al.* (1997a).

3. Data-Based Mechanistic (DBM) Modelling

Previous publications (Beck & Young, 1975; Whitehead & Young, 1975; Young, 1978, 1983, 1992, 1993, 1998a,b; Young & Minchin, 1991; Young & Lees, 1993; Young & Beven, 1994; Young *et al.*, 1996; Young & Pedregal, 1998, 1999a; Young and Parkinson, 2002) map the evolution of the DBM philosophy and its methodological underpinning in considerable detail, and so it will suffice here to merely outline the main aspects of the approach.

The main stages in DBM model building are as follows:

1. The important first step is to define the objectives of the modelling exercise and to consider the type of model that is most appropriate to meeting these objectives. For instance a complex spatio-temporal catchment model to be used for the purpose of ‘what-if’ studies within a planning context is not necessarily a suitable vehicle for real-time flow forecasting, and vice versa. As a result, the prior assumptions about the form and structure of this model are kept at a minimum in order to avoid the prejudicial imposition of untested

perceptions about the nature and complexity of the model needed to meet the defined objectives.

2. The next step in DBM modelling depends upon the circumstances of the Study. Prior to the acquisition of data (or in situations of insufficient data), it is advisable and often essential to develop a simulation model that represents the system in a physically meaningful manner and satisfies the defined objectives. It is also necessary to evaluate the sensitivity of this model to uncertainty and develop a reduced order, ‘dominant mode’ model (see previous section) that captures the most important aspects of its dynamic behaviour.
3. Subsequent the acquisition of sufficient data, an appropriate model structure is identified and its parameters estimated by a process of relatively objective statistical inference applied directly to the time-series data and based on a given general class of linear and nonlinear stochastic models, such as differential equations or their discrete-time equivalents. This stage normally exploits recursive estimation methods (e.g. Young, 1984) and can involve both non-parametric and parameteric estimation, as discussed in later sections of the Chapter.
4. Regardless of whether the model is identified and estimated in linear or nonlinear form, it is only accepted as a credible representation of the system if, in addition to explaining the data well, it also provides a description that has direct relevance to the physical reality of the system under study. This is a most important aspect of DBM modelling and differentiates it from more classical statistical modelling methodology and ‘neural’ methods of black-box modelling. If necessary, the DBM model obtained at this stage should be reconciled with the dominant mode version of the simulation model considered in 2. above.
5. Finally, the estimated model is tested in various ways to ensure that it is ‘conditionally valid’ in the sense discussed in the next section. This involves standard statistical diagnostic tests for stochastic, dynamic models, including analysis which ensures that the nonlinear effects have been modelled adequately (e.g. Billings & Voon, 1986), as well as exercises in predictive validation and stochastic sensitivity analysis.

One aspect of the above DBM approach which differentiates it from alternative deterministic top-down approaches is its inherently stochastic nature and exploitation of powerful statistical methods for the identification and parameter estimation. This means that the uncertainty in the estimated model is always quantified and this information can then be utilized in various ways. For instance, it allows for the application of Monte Carlo-based uncertainty and sensitivity analysis, as well as the use of the model in statistical forecasting and data assimilation algorithms, such as the Kalman Filter. The uncertainty analysis is particularly useful because it is able to evaluate how the covariance properties of the parameter estimates affect the probability distributions of physically meaningful, derived parameters, such as residence times and partition percentages in parallel hydrological pathways (see e.g. Young, 1992, 1999a and the example below).

4. Statistical Identification, Estimation and Validation

The statistical approach to modelling assumes that the model is stochastic: in other words, no matter how good the model and how low the noise on the observational data happens to be, a certain level of uncertainty will remain after modelling has been completed. Consequently, full stochastic modelling requires that this uncertainty, which is associated with both the model parameters and the stochastic inputs, should be quantified in some manner as an inherent part of the modelling analysis. This statistical approach involves three main stages: *identification* of an appropriate, identifiable model structure; *estimation* (optimization, calibration) of the parameters that characterize this structure, using some form of estimation or optimization; and *conditional predictive validation* of the model on data sets different to those used in the model identification and estimation. In this section, we consider these three stages in order to set the scene for the later analysis. This discussion is intentionally brief, however, since the topic is so large that a comprehensive review is not possible in the present context.

(a) Structure and Order Identification

Identification in a statistical context normally means the data-based inference of the most appropriate model order, as defined in dynamic system terms, although the model structure itself can be the subject of the analysis if this is also considered to be ill-defined. Most importantly, the nature of linearity and nonlinearity in the model is not assumed *a priori* (unless there are good reasons for such assumptions based on previous data-based modelling studies) but is identified from the data using non-parametric and parametric statistical estimation methods.

Within the hydrological ‘top-down’ context, linear system identification analysis is related directly to problems such as the definition of how many storage zones (conceptual ‘storages’ or ‘buckets’) are required to characterize the data *at the scale of interest*; and how these sub-models are interconnected (i.e. in series, parallel or feedback arrangements). It must be stressed, however, that such problems arise mainly from the specification of the *dynamic* model order (i.e. the order of the differential equations that are used to describe the major rainfall-flow dynamics; or equivalently, here, the number of storage zones). So a parsimonious model, in this important dynamic sense, is one that has a lowest dynamic order that is consistent with the information content in the data and whose parameters are statistically significant.

Of course, the DBM model may well have *other* parameters that are not associated primarily with the dynamic order of the model and are not so important in identifiability terms: for instance, coefficients that parameterize any nonlinearity in the system (see below). Here again, however, the presence of such parameters in the model should be justified by whether or not they are statistically significant. The statistical significance of parameter estimates can be evaluated by conventional statistical tests or, in these days of the fast digital computer, by Monte Carlo simulation and sensitivity analysis (see e.g. chapter 6 in Saltelli *et al.*, 2000 and chapter 7 in Beven, 2001).

There are a variety of statistical methods for identifying model order. Fitting cri-

teria, such as the coefficient of determination† (R_T^2 : see later) based on the simulated model errors, can be very misleading if used on their own, since over-parameterized models can ‘over-fit’ the data. In general, therefore, it is necessary to exploit some specific order identification statistics, such as: the correlation-based statistics popularized by Box & Jenkins (1970); the well known Akaike Information Criterion (AIC: Akaike, 1974); and the YIC criterion proposed by the present author (Young, 1989). In all cases, the objective is to avoid over-parametrization by identifying a model structure and order that explains the data well within a minimal parametrization: i.e. ‘parsimonious models’ (Box & Jenkins, 1970). The time series methods used for model order identification in the present report are outlined in Young and Lees (1993), Young and Beven (1994), Young et al. (1996) and Young and Parkinson (2002).

In DBM modelling, identification also includes the initial data-based estimation of the location and nature of any significant nonlinear phenomena that need to be included in the model. Fortunately, the recursive estimation procedures that figure so strongly in DBM modelling allow for the estimation of any significant parameter variation in the model and this can provide information on the presence of nonstationarity and nonlinearity in the system dynamics. Here, the model parameters are estimated by the application of an approach to *Time Variable Parameter* (TVP) or *State-Dependent Parameter* (SDP) estimation based on recursive Fixed Interval Smoothing (FIS): e.g. Bryson & Ho (1969); Young (1984, 1999b, 2000). In the SDP case, the temporal variation in the parameters is further related to the variation of other measured variables or ‘states’ that characterize the system, thus identifying the presence of nonlinear behaviour. In effect, the FIS algorithm provides a method of non-parametric SDP estimation, with the estimates defining a graph of the SDP against the associated state variable, which then defines the nature of the nonlinearity (see Young, 1993; Young & Beven, 1994; Young, 1998a, 2000, 2001a; Young et al., 2001). This approach to SDP estimation is illustrated in the later practical example

(b) *Estimation (Optimization or Calibration)*

Once the model structure and order have been identified, the parameters that characterize this structure need to be estimated in some manner. There are many automatic methods of estimation or optimization available in this age of the digital computer, from the simplest, deterministic procedures, usually based on the minimization of least squares cost functions; to more complex numerical optimization methods based on statistical concepts, such as Maximum Likelihood (ML). In general, the latter are more restricted, because of their underlying statistical assumptions, but they provide a more thoughtful and reliable approach to statistical inference; an approach which, when used correctly, includes the associated statistical diagnostic tests that are considered so important in statistical inference.

In DBM modelling, if the model is identified as predominantly linear or piecewise linear, then the constant parameters that characterize the identified model structure are estimated using advanced methods of statistical estimation for linear *Transfer Function* (TF) models. As shown in Appendix A, TF models are

† often termed the ‘Nash-Sutcliffe efficiency’ in the hydrological literature (Nash & Sutcliffe, 1970)

simply representations of linear differential equations or their discrete-time (sampled data) equivalents. Here, the preferred methods are the *Refined Instrumental Variable* (RIV/SRIV) algorithms, which provide a robust approach to model identification and estimation that has been well tested in practical application to hydrological systems over many years. Although based on ML estimation concepts, these algorithms are more robust to assumptions about the nature of the noise and uncertainty affecting the system. Full details of the methods are provided in Young & Jakeman (1979, 1980); Jakeman & Young (1979); Young, (1984, 1985). They are also outlined in Young & Beven (1994), Young *et al.*(1996) and Young and Parkinson (2002).

If nonlinear phenomena have been detected and identified in stage (a), the non-parametric, state dependent relationships are normally parameterized in a finite form and the resulting nonlinear model is estimated using some form of numerical optimization, such as nonlinear least squares or ML based on prediction error decomposition (Schweppe, 1965). In the present flow forecasting context, this approach to nonlinear estimation is required only to define the nature of the effective rainfall nonlinearity, which appears at the input to the model, as described in subsequent sections of this Chapter.

(c) *Conditional Predictive Validation*

Validation is a complex process and even its definition is controversial. Some academics (e.g. Konikow & Brederhoeft, 1992, within a ground-water context; Oreskes *et al.*, 1994, in relation to the whole of the earth sciences) question even the possibility of validating models. To some degree, however, these latter arguments are rather philosophical and linked, in part, to questions of semantics: what is the ‘truth’? What is meant by terms such as validation, verification and confirmation? etc. Nevertheless, one specific, quantitative aspect of validation is widely accepted; namely predictive validation, in which the predictive potential of the model is evaluated on data other than that used in the identification and estimation stages of the analysis.

Predictive validation implies that, on the basis of the new measurements of the model inputs (e.g. rainfall) from the validation data set, the model produces flow predictions that are acceptable within the uncertainty bounds associated with the model. Note this stress on the question of the inherent uncertainty in the estimated model: one advantage of statistical estimation, of the kind considered in this chapter, is that the level of uncertainty associated with the model parameters and the stochastic inputs is quantified in the time series analysis. Consequently, the modeller should not be looking for perfect predictability (which no-one expects anyway) but predictability which is consistent with the quantified uncertainty associated with the model.

5. The DBM Catchment Model

Within the catchment modelling context, DBM models are of two types: the *Non-linear Rainfall-Flow Model*; and the *Flow Routing Model*, which may be linear or nonlinear, depending on the nature of the catchment and the flow or stage gauging. The complete model used in flood forecasting and warning applications is com-

prised of both types linked in a manner that reflects the physical nature of the catchment under study. In this paper, however, we concentrate almost completely on the rainfall-flow component, with only a brief reference to flow routing. It must be emphasized, however, that this is not because flow routing is unimportant in real-time flood forecasting. It is simply that the advances reported in this Chapter relate almost entirely to rainfall-flow modelling.

(a) *The Rainfall-Flow Component*

The first step in DBM modelling is the consideration of the objectives. In this case, it will be assumed that this is limited to obtaining a model which explains the rainfall-flow data well on an hourly or daily basis at the *whole catchment scale*; and, at the same time, is capable of reasonable mechanistic interpretation combined with an ability to perform well in a flood forecasting/warning context. Note that this emphasis on the ‘catchment scale’ is very important because the hydrological significance and interpretation of the rainfall-flow models developed below all relate to catchment scale characteristics, such as storage and flow partitioning. These models do not relate directly to more detailed characteristics such as flow paths in the field, analysis of soil depths etc. Note also the allusion to the ‘rainfall-flow’ relationship, rather than the use of the more conventional ‘rainfall-runoff’ terminology. This is to emphasize that, as discussed below, the models considered here predict both storm runoff and base-flow, which are interpreted as the major components of the *total* gauged flow.

Based on these objectives, the most obvious and physically meaningful model form in this hydrological context is a continuous-time, differential equation (or set of equations). Such a model is consistent, for example, with the normal formulation of conservation equations and many conventional hydrological models: e.g. conceptual models of serial and parallel connected nonlinear ‘buckets’ or ‘storages’, as discussed, for instance, in the top-down modelling of Jothityangkoon *et al.* (2001)[†] (see the description of such models in Young, 2003). However, when dealing with discrete-time, sampled data, it is often convenient to consider modelling in terms of the discrete-time equivalent of the differential equation, the discrete-time TF. Appendix A provides some background on TF models and shows the link between continuous and discrete-time TF models. In the rest of the Chapter, we concentrate on modelling in discrete-time terms but the treatment is very similar for continuous-time differential equation models estimated from discrete-time data, as discussed in Young (2004).

Using the discrete-time TF model form of equation (A11) in Appendix A, previous DBM modelling of rainfall-flow data based on SDP estimation (see the references in the previous section) has confirmed many aspects of earlier hydrological research and identified the nonlinear DBM model structure shown in figure 1[‡]. Here, the two components of the TF model are the linear component, which models the basic, underlying, hydrograph behaviour; and the nonlinear component, which mod-

[†] although these authors discuss modelling at annual, monthly and daily scales, the conceptual arguments are similar.

[‡] This model is similar in concept to the variable gain factor model suggested by Ahsan & O’Connor (1993), although its identification, estimation and implementation is quite different.

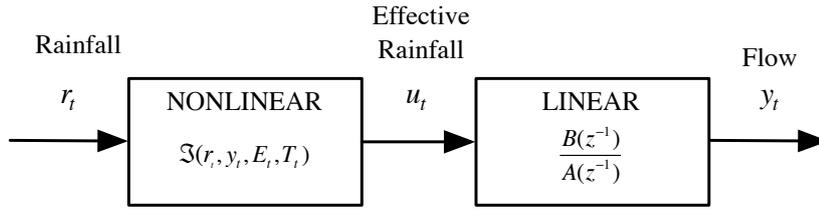


Figure 1. Block diagram of the generic DBM rainfall-flow model.

els the relationship between the *measured rainfall* r_t and the *effective rainfall* u_t , so controlling the magnitude of the hydrograph contribution through time.

The resulting DBM model has the form (see Appendix A):

$$y_t = \frac{B(z^{-1})}{A(z^{-1})} u_{t-\delta} + \xi_t \quad (1a)$$

In these equations, z^{-1} is the backward shift operator, i.e., $z^{-r} y_t = y_{t-r}$, while $A(z^{-1})$ and $B(z^{-1})$ are *constant coefficient polynomials* in z^{-1} of the following form:

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_m z^{-m}. \end{aligned} \quad (1b)$$

The term δ is a pure time delay, measured in sampling intervals, which is introduced to allow for any temporal (advective) delay that may occur between the incidence of a change in u_t and its first effect on y_t . It is assumed that y_t is also affected by an additive noise component ξ_t that represents the effects of all noise inputs, such as measurement inaccuracy, un-measured stochastic disturbances and modelling errors.

The structure (order) of the TF model (1a) is defined by the triad $[n \ m \ \delta]$ and this is normally identified from the data during the identification and estimation of the model, based on historical rainfall-flow data. This order is normally low, with $n \leq 3$, $m \leq 3$; while the value of δ is defined by the nature of the catchment and the location of the measurement devices, so its range is more difficult to define *a priori*. The general TF model form $B(z^{-1})/A(z^{-1})$ defines the input-output relationship between u_t and y_t and its unit impulse response is a scaled version of the underlying *Unit Hydrograph*. But, as we see later, it can also be decomposed into a parallel connection of lower order processes. This decomposition not only makes the physical interpretation of the TF more transparent, it can also improve its performance in forecasting terms when implemented within a flood forecasting system.

The nonlinear component in figure 1 takes the general form:

$$u_t = \mathcal{F}(r_t, y_t, E_t, T_t) \quad (1c)$$

where $\mathcal{F}(r_t, y_t, E_t, T_t)$ denotes an unknown, nonlinear functional relationship defining the unobserved catchment storage state s_t (or, as we shall see later, some surrogate for this state) considered as a function of potentially important variables that may affect or be related to catchment storage. In addition to the rainfall r_t , this function may involve other relevant measured variables, such as the temperature T_t

(or some function of this, such as the mean monthly temperature T_m), the potential evaporation E_t and the flow y_t ; all of which could help to define the changes in soil moisture and storage if they are available.

Typically, the form of the nonlinearity $\mathcal{F}(\cdot)$ is initially identified from the rainfall-flow data through SDP estimation in non-parametric (graphical or ‘look-up’ table) form, without any prior assumptions about its nonlinear nature. This is then parameterized in some simple manner: for example, in Young (1993), Young & Beven (1994) and Young & Tomlin (2000), $\mathcal{F}(\cdot)$ is defined as a power law cy_t^γ in the flow y_t , with the power law exponent γ estimated from the data. In this case, the complete rainfall-flow model takes the very simple form:

$$y_t = \frac{B(z^{-1})}{A(z^{-1})}u_{t-\delta} + \xi_t \quad u_t = cy_t^\gamma r_t \quad (1d)$$

The fact that the nonlinearity here $\mathcal{F}(\cdot)$ is a function of the flow y_t seems very strange at first sight but the reason for this is discussed later in section 6. The attraction of this SDP estimation approach is that $\mathcal{F}(\cdot)$ is inferred from the rainfall-flow data and not assumed *a priori*, as in conceptual HMCs such as the Bedford-Ouse and IHACRES models, so leaving less room for unjustified over-confidence in the hypothetical definition of the nonlinear model form.

The DBM model, even in the simple form of equations (1a) to (1d), appears to have wide application potential. In addition to rivers in Australia (e.g. Young *et al.*, 1997a,b) and the USA (Young, 2001b), it has been combined with an adaptive gain updating scheme in the parameter-adaptive Dumfries flood warning system (Lees *et al.*, 1994), which has been operating successfully without major modification since 1991; and it is embedded within the Kalman Filter to provide a *State-Adaptive* forecasting system for the River Hodder in NW England (Young & Tomlin, 2000); and both state and parameter adaptive systems in (Young, 2002) for the River Hodder and Romanowicz *et al.* (2004) for the River Severn, England .

(b) The Flow Routing Component

The generic flow (channel) routing model is much simpler than the rainfall-flow model since it is now widely accepted that linear TF models are often (but not always) adequate for the representation of flow dynamics in river systems. The discrete-time routing model for a single stretch of river consists of a serial connection of channel storage elements, each of which has the general form:

$$y_t^i = \frac{B_i(z^{-1})}{A_i(z^{-1})}y_{t-\delta_i}^{i-1} + \xi_t^i \quad i = 1, 2, \dots, nr \quad (1e)$$

where nr is the number of reaches and the i superscript denotes the reach number. Once again, it is assumed that the output of the routing model is affected by noise, here denoted by ξ_t^i . Equation (1e) can be considered as the discrete-time equivalent of continuous-time differential equation storage equations (see Appendix A). Normally, each of these elements is only first or second order (as defined by statistical identification and estimation based on the up-stream and down-stream flow data). The complete catchment routing model will consist of models such as this for the main river channels and all their tributaries within the catchment,

connected accordingly; and it can involve any other measured in-flows as additions, inserted at appropriate nodal locations. The model will receive inputs from the rainfall-flow models discussed above and, in examples such as the Dumfries flood warning model, from flow gauges far upstream which provide advance warning of impending flow changes. A typical early example of such a model is that used for studies of the Bedford-Ouse river system (Whitehead *et al.*, 1976); more recent examples are the simple River Wyre model (Young, 1986); the much more spatially complex Dumfries model (Lees *et al.*, 1994) and other models discussed in Cluckie (1993).

Note that we are restricting attention here to TF-based flow routing: this is not, of course, the only form of flow routing and other approaches are often utilized, although most of these can be considered in TF terms if this is desired (e.g. the ‘kinematic wave’ model used in the *Thames Catchment Model*: see Greenfield, 1984; Moore & Jones, 1978). As in the case of the rainfall-flow models, flow routing TF model parameters are normally obtained by the analysis of historical flow records using similar statistical identification and estimation methods to those used in the rainfall-flow example above.

6. Physical Interpretation of the DBM Model

As we have stressed, an important aspect of DBM modelling is that the model can be interpreted in physically meaningful terms. In this regard, let us consider the model (1d) which, as we shall see, is the model used in the Canning River example considered later in section 8. Here, at first sight, the relationship $u_t = c.f(y_t).r_t$ appears hard to justify in physical terms. However, it should not be taken literally and interpreted as saying that the effective rainfall is physically a function of flow, which is physically impossible. Rather, the measured flow y_t is effectively acting here as an objectively identified *surrogate* for the catchment storage s_t . This seems sensible from a hydrological standpoint, since flow is clearly a function of the catchment storage and its pattern of temporal change is likely to be similar. So, the nonlinear function as a whole is similar in its motivation to that used in the Bedford Ouse and IHACRES conceptual HMC models and can be justified similarly in physical terms.

The delayed effective rainfall $u_{t-\delta}$ provides the input to the linear TF model component (1a). If this TF is greater than first order and characterized by real eigenvalues (the roots of the $A(z^{-1})$ polynomial), as it normally will be, then the TF can be decomposed into a parallel pathway form, with first order storage equations in each pathway (see the discussion on the physical interpretation of parallel TF models in Wallis *et al.*, 1989; Jakeman *et al.*, 1990; Young, 1992, 1993; Young & Beven, 1994; and Lees, 2000a,b). From this decomposition, it is possible to compute the residence times (time constants); the advective time delays; the percentage partition of the flow down each of the storage pathways; and even the changing volumes associated with these pathways, all with obvious physical significance. When dealing with hourly data, there are usually two such pathways with very different dynamic characteristics. In the case of daily data, there can be three pathways, one of which represents an ‘instantaneous’ response: i.e. a flow effect that arises within the same day of the effective rainfall occurrence. For example, in the case of the Canning River example, the best identified DBM model has a [2 3 0] TF that, so

that the model can be decomposed to the form:

$$y_t = 0.06u_t + \frac{0.171z^{-1}}{1 - 0.661z^{-1}}u_t + \frac{0.026z^{-1}}{1 - 0.942z^{-1}}u_t + \xi_t \quad (2)$$

Here, the first component on the right hand side of the equation is the instantaneous effect which accounts for only 5.9% of the flow. The second component is the fast pathway effect and this accounts for most of the flow, at 49.5%, with a residence time of 2.42 days. The final component is the slow pathway effect, which accounts for 44.6% of the flow and has a residence time of 16.73 days.

Given these derived model parameters, the most obvious physical interpretation of the DBM is that the effective rainfall affects the river flow via two main pathways. First, the initial rapid rise in the hydrograph derives from the quick-flow pathway, probably as the aggregate result of the many surface processes active in the catchment. And the long, elevated tail in the recession of the hydrograph arises from the slow-flow component, most likely the result of water displacement (probably of old water) from the storage within the groundwater system. Note that the estimate of the flow contribution of this slow-flow component is also practically useful in other ways: it provides a relatively objective estimate of the total base-flow in the river and, as such, can be utilized for base flow quantification and removal, if this is required, as suggested by Jakeman *et al.* (1990). This contrasts with the classical unit hydrograph methods, where the base-flow has to be removed rather subjectively.

Finally, it must be emphasized that the estimated TF and its decomposition are stochastic objects and so the uncertainty that is inherent in their derivation needs to be taken into consideration when interpreting the model in physically meaningful terms, as illustrated in the later Canning River example.

7. Data Assimilation: the Recursive Kalman Filter, State and Parameter-Adaptive Forecasting

Within a flood forecasting context, a catchment model based on rainfall-flow and flow routing TF models should not be considered as an end in itself: rather, it is a major component of a *data assimilation* system that collects data from remote sensors within the catchment and ‘blends’ these data with the model in a statistical manner to produce acceptable forecasts into the future. In this situation, the DBM model identified and estimated from the data with the objective of explaining the data may well not provide the best model for forecasting. The objective of forecasting is not simply to explain the data, but rather to maximize the forecasting performance over a specified lead time into the future. If we consider the Canning River model (2), for instance, it has no input time delay ($\delta = 0$) so, in order to forecast even one-day-ahead, it is necessary to also forecast the rainfall one-day-ahead which, in itself, is quite a difficult task. On the other hand, if we are able to introduce a time delay of one day into the model and still explain the data well, then there is no need for a rainfall forecast and the resulting flow forecasts are likely to be superior to those using the originally optimized model.

For illustrative purposes, let us consider the model formulation in terms of a single, second order [2 2 1] rainfall-flow model of the Canning, where there is a one

day delay ($\delta = 1$) rather than the zero delay in (2). The decomposed form of this estimated model is as follows:

$$y_t = \frac{0.185z^{-1}}{1 - 0.679z^{-1}}u_{t-1} + \frac{0.024z^{-1}}{1 - 0.946z^{-1}}u_{t-1} + \xi_t \quad (3)$$

As we shall see later in section 8, although this is not the best identified model, it still explains 94.9% ($R_T^2 = 0.949$) of the Canning flow series over a two year period, which is almost indistinguishable from the model (2), where $R_T^2 = 0.950$.

The model (3) could be used directly as a basis for real-time data assimilation and forecasting but, as we shall see, real-time updating is made much more straightforward and flexible if it is converted into a stochastic state space form so that it can be embedded within a *Kalman Filter* (KF) state estimation algorithm. In the simplest situation, where the ξ_t is a white noise process e_t , with variance σ^2 , the most obvious stochastic state space form of (3) is:

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \mathbf{G}u_{t-1} + \boldsymbol{\zeta}_t \quad (4a)$$

$$y_t = \mathbf{h}^T \mathbf{x}_t + e_t \quad (4b)$$

where:

$$\mathbf{F} = \begin{bmatrix} 0.679 & 0 \\ 0 & 0.946 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 0.185 \\ 0.024 \end{bmatrix} \quad \boldsymbol{\zeta}_t = \begin{bmatrix} \zeta_{1,t} \\ \zeta_{2,t} \end{bmatrix} \quad \mathbf{h}^T = [\ 1 \ 1 \] \quad (4c)$$

In this manner, the state variables are defined as the unobserved (hidden or latent) quick and slow components of the flow, as defined by the decomposed, first order TF relationships; and the output or ‘observation’ equation (4b) combines these to form the complete flow output. For simplicity, the stochastic ‘system inputs’ $\zeta_{1,t}$ and $\zeta_{2,t}$ are assumed to be zero mean, white noise processes. They are introduced to allow for the inevitable uncertainty in the definition of the parallel pathway dynamics and are an important aspect of this ‘state-adaptive’ approach to forecasting.

For flow forecasting purposes, this state space model is used as the basis for the implementation of the following, recursive, KF state estimation and forecasting algorithm:

***A priori* prediction:**

$$\begin{aligned} \hat{\mathbf{x}}_{t|t-1} &= \mathbf{F}\hat{\mathbf{x}}_{t-1} + \mathbf{G}u_t \\ \mathbf{P}_{t|t-1} &= \mathbf{F}\mathbf{P}_{t-1}\mathbf{F}^T + \sigma^2\mathbf{Q}_r \\ \hat{y}_{t|t-1} &= \mathbf{h}^T \hat{\mathbf{x}}_{t|t-1} \end{aligned}$$

***A posteriori* correction:**

$$\begin{aligned} \hat{\mathbf{x}}_t &= \hat{\mathbf{x}}_{t|t-1} + \boldsymbol{\Pi}_t \cdot \{y_t - \hat{y}_{t|t-1}\} \\ \boldsymbol{\Pi}_t &= \mathbf{P}_{t|t-1}\mathbf{h}[\sigma^2 + \mathbf{h}^T\mathbf{P}_{t|t-1}\mathbf{h}]^{-1} \\ \mathbf{P}_t &= \mathbf{P}_{t|t-1} - \boldsymbol{\Pi}_t\mathbf{h}^T\mathbf{P}_{t|t-1} \\ \hat{y}_t &= \mathbf{h}^T \hat{\mathbf{x}}_t \end{aligned}$$

In these equations, \mathbf{P}_t is the error covariance matrix associated with the state estimates; and \mathbf{Q}_r is the 2×2 *Noise Variance Ratio* (NVR) matrix defined below.

(a) *State Adaption*

In the above KF equations, the DBM model parameters are known initially from the model identification and estimation analysis based on the estimation data set. However, by embedding these model equations within the KF algorithm, we have introduced additional, unknown parameters, normally termed ‘hyper-parameters’ to differentiate them from the model parameters[†]. In this example, these hyper-parameters are the elements of the *Noise Variance ratio* (NVR) matrix \mathbf{Q}_r and, in practical terms, it is normally sufficient to assume that this is purely diagonal in form. These two diagonal elements are defined as $NVR_i = \sigma_{\zeta_i}^2 / \sigma^2$, $i = 1, 2$: they specify the nature of the stochastic inputs to the state equations and so define the level of uncertainty in the evolution of each state (the quick and slow flow states respectively) relative to the measurement uncertainty. The inherent state adaption of the KF arises from the presence of the NVR parameters since these allow the estimates of the state variables to be adjusted to allow for presence and effect of the unmeasured stochastic input disturbances.

Clearly, the NVR hyper-parameters have to be estimated in some manner on the basis of the data. One well known approach is to exploit *Maximum Likelihood* (ML) estimation based on *Prediction Error Decomposition* (see Schweppe, 1964; Young, 1999b). Another, is to assume that *all* the parameters of the state space model (8a,b) are unknown and re-estimate them by minimizing the variance of forecasting errors over the specified forecasting interval. In effect, this optimizes the memory of the recursive estimation and forecasting algorithm (Young & Pedregal, 1999b) in relation to the rainfall-flow data. In this numerical optimization, the multi-step-ahead forecasts $\hat{y}_{t+f|t}$, where f is the forecasting horizon, are obtained by simply repeating the prediction step in the algorithm f times, without intermediate correction. The main advantage of this latter approach is, of course, that the integrated model-forecasting algorithm is optimized directly in relation to the main objective of the forecasting system design; namely the minimization of the multi-step prediction errors.

(b) *Parameter Adaption*

Although the parameters and hyperparameters of the KF-based forecasting engine can be optimized in the above manner, we cannot be sure that the system behaviour may not change sufficiently over time to require their adjustment. In addition, it is well known that the measurement noise e_t is quite highly heteroscedastic: i.e. its variance can change quite radically over time, with much higher variance occurring during storm events. For these reasons, it is wise to build some form of parameter adaption into the forecasting algorithm.

(i) *Gain Adaption*

It is straightforward to update *all* of the parameters in the rainfall-flow model since all the estimation algorithms can be implemented in a recursive form that allows for sequential updating and the estimation of time-variable parameters (Young,

[†] Of course this differentiation is rather arbitrary since the model is inherently stochastic and so these parameters are simply additional parameters introduced to define the stochastic inputs to the model when it is formulated in this state space form

1984). However, this adds complexity to the final forecasting system and previous experience suggests that a simpler solution, involving a scalar gain adaption is often sufficient. This is the approach that has been used successfully for some years in the Dumfries flood warning system (Lees *et al.*, 1994) and it involves the recursive estimation of the gain $g(k)$ in the following relationship:

$$y_t = g_t \hat{y}_t + \epsilon_t \quad (5a)$$

where ϵ_t is a noise term representing the lack of fit and, in the case of the model (1d),

$$y_t = \frac{\hat{B}(z^{-1})}{\hat{A}(z^{-1})} u_{t-\delta} \quad u_t = c y^{\hat{y}} r_t \quad (5b)$$

where the ‘hats’ indicate estimated values. In other words, the time variable scalar gain parameter g_t is introduced so that the model gain can be continually adjusted to reflect any changes in the steady state (equilibrium) response of the catchment to the effective rainfall inputs.

The associated recursive estimation algorithm for g_t takes the usual *Recursive Least Squares* (RLS) form in the case where g_t is assumed to vary stochastically as a *Random Walk* (RW) process (e.g. Young, 1984):

$$p_{t|t-1} = p_{t-1} + q_g \quad (5c)$$

$$p_t = p_{t|t-1} - \frac{p_{t|t-1}^2 \hat{y}_t^2}{1 + p_{t|t-1} \hat{y}_t^2} \quad (5d)$$

$$\hat{g}_t = \hat{g}_{t-1} + p_t \hat{y}_t \{y_t - \hat{g}_{t-1} \hat{y}_t\} \quad (5e)$$

where \hat{g}_t is the estimate of g_t ; while q_g is the NVR defining the stochastic input to the RW process, the magnitude of which needs to be specified or optimized (see later). The adapted forecast is obtained by simply multiplying the initially computed forecast by \hat{g}_t . Note that gain adaption of this kind is quite generic and can be applied to *any* model, not just those discussed here.

(ii) Variance Adaption

To allow for the heteroscedasticity in e_t , it is necessary to recursively estimate† its changing variance σ_t^2 . Although a logarithmic transform might suffice, a superior approach is to use the transformation is $c_t = \log(\chi_t^2) + \lambda$, where the stochastic process χ^2 defined by,

$$\chi_m^2 = (e_{2m-1}^2 + e_{2m}^2)/2 \quad m = 1, \dots, N/2 \quad (6a)$$

in which $\lambda = 0.57722$ is the Euler constant. This is motivated by Davis & Jones (1968), who showed that c_t has a theoretical distribution that is almost normal. As a result, an estimate \hat{h}_t of the transformed variance can be obtained from the following recursive least squares algorithm (*cf* the above RLS algorithm for \hat{g}_t),

† a non-recursive ML formulation of this heteroscedasticity problem is given by Sorooshian (1985)

where this time it is c_t that is assumed to vary stochastically as a RW process:

$$p_{t|t-1} = p_{t-1} + q_h \quad (6b)$$

$$p_t = p_{t|t-1} - \frac{p_{t|t-1}^2}{1 + p_{t|t-1}} \quad (6c)$$

$$\hat{h}_t = \hat{h}_{t-1} + p_t \{c_t - \hat{h}_{t-1}\} \quad (6d)$$

An estimate $\hat{\sigma}_t^2$ of σ_t^2 can then be obtained as $\hat{\sigma}_t^2 = \exp(\hat{h}_t - \lambda)$.

(iii) *Hyper-parameter estimation*

The above RLS estimation algorithms are, in fact, very simple examples of the KF and so it is necessary to estimate the hyper-parameters (here q_g and q_h) in some manner. Their joint estimation with the KF hyper-parameters is straightforward. However, a simpler, heuristic approach derives from the fact that q_g and q_h control the memory of their respective RLS estimation algorithms and the associated smoothing of the estimate (e.g. Young, 1984). Consequently, since q_g and q_h are scalar values, it is not difficult to optimize them manually to yield the best multi-step-ahead forecasts.

8. Illustrative Practical Example: The Ephemeral Canning River in South Western Australia

Figure 2 shows a portion of effective rainfall $u(t)$, flow $y(t)$ and air temperature $T(t)$ data from the Canning River at Glen Eagle in SW Australia over the period 23rd March 1985 to 26th February 1987. The Canning is a 544 km² benchmark catchment whose discharge is dominated by zero flows, with zero flow periods occupying more than half of the recorded period. The rainfall is Winter dominated, with the Winter 4 months receiving approximately 70% of the total annual rainfall. These data have been analysed previously by Young *et al.* (1997a) and Ye *et al.* (1998). The latter reference uses a modified version of the IHACRES model. The present DBM modelling results are similar to those in the former reference but the analysis has been improved in various ways, so the results are somewhat different.

(a) *DBM Modelling*

In this example, the time series data are plentiful so the first stage in DBM modelling is model structure identification, based on the estimation data set in Figure 2. Initial linear identification, using the RIV estimation algorithm, suggests a [2 3 0] discrete-time TF model but its explanation of the data is poor, implying that a nonlinear model is required, as would be expected on physical grounds. TVP estimation confirms this and previous experience, showing that a time variable parameter model is able to explain the data well. As a result, the identification proceeds further using SDP estimation, as outlined in previous sections. Figure 3 shows the non-parametric estimation results obtained in this manner.

The estimated shape of the effective rainfall nonlinearity is similar to that obtained in most previous rainfall-flow modelling studies and the DBM model based

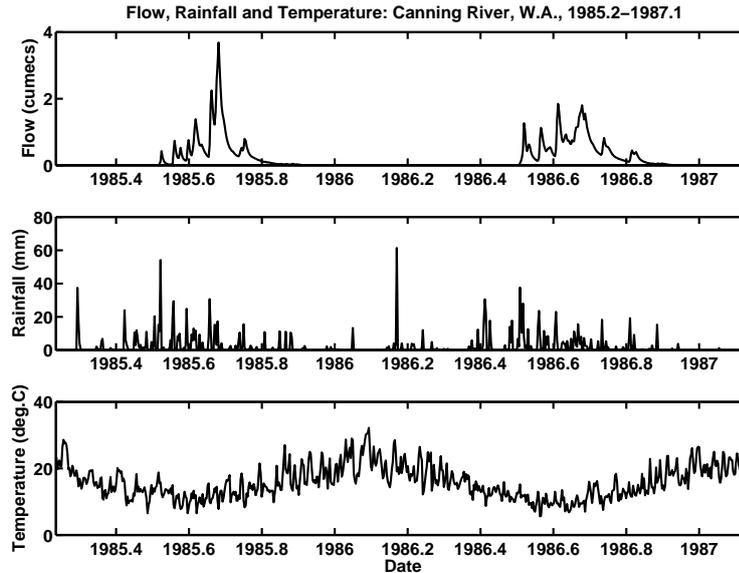


Figure 2. Two years of daily flow (upper panel), rainfall (middle panel) and air temperature (lower panel) for the ephemeral Canning River at Glen Eagle in South Western Australia.

on these non-parametric results explains 94.7% of the flow data ($R_T^2 = 0.947$). Again, based on previous experience, a power law seems to be reasonable contender for parameterization of the SDP nonlinearity in Figure 3, although it is clearly not the only possibility. Parametric estimation proceeds with this definition and yields the following parameter estimates:

$$\begin{aligned} \hat{a}_1 &= -1.6031(0.012); & \hat{a}_2 &= 0.6228(0.011); & \hat{\gamma} &= 0.823(0.09); \\ \hat{b}_1 &= 0.0601(0.005); & \hat{b}_2 &= 0.100(0.010); & \hat{b}_3 &= -0.1405(0.006); \\ \hat{\beta}_1 &= 0.171; & \hat{\alpha}_1 &= 0.661; & \hat{\beta}_2 &= 0.026; & \hat{\alpha}_2 &= 0.942; \end{aligned}$$

The decomposed form of this model has been presented previously in equation (2). Note that the derived parameters $\alpha_i, \beta_i, i = 1, 2$ associated with the decomposed model have no specified uncertainty bounds because they are not estimated directly. However, we consider the uncertainty in such derived parameters later. It will be noted also that, although the parametric estimate in Figure 3 (dash-dot line) does not entirely reflect the shape of the non-parametric estimate (solid line), it does lie mostly within the associated 95% confidence region. Consequently, given that the parametric estimate is also uncertain (bounds not shown to avoid confusion), it is clear that the two estimates are statistically compatible. Of course, the parametric estimate is more statistically efficient.

The simulated output of the estimated DBM model is shown as the full line in Fig. 4: it clearly explains the data (solid points) well, consistent with the high $R_T^2 = 0.958$. The decomposition of the model has been discussed above in section 6 and Figure 5 shows the estimated flows through the different pathways, all plotted to the same scale. We see that the instantaneous effect (within one day) plotted in the top panel is very small, as would be expected. The quick and slow components

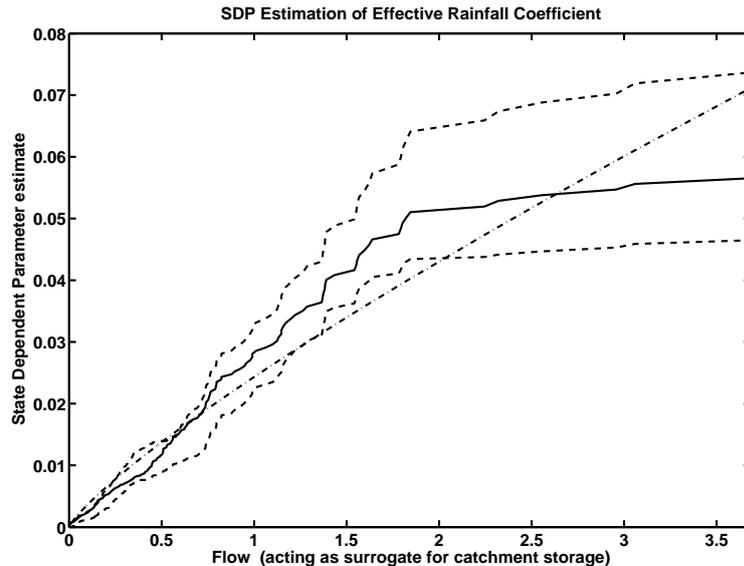


Figure 3. SDP non-parametric estimation of the effective rainfall nonlinearity: SDP estimate (full line) and associated 95% confidence bounds (dashed lines). Also shown (dash-dot line) is the parametric estimate of the nonlinearity based on a power law parameterization, as obtained at the final parameter estimation phase of the analysis

explain much more of the flow response and it is clear that it is the latter slow pathway that leads to the extended ‘tail’ on the associated hydrograph and can be associated with the slowly changing ‘baseflow’ in the river.

The residual noise in the estimated DBM model, ξ_t , is quite complex and heteroscedastic. On the basis of the AIC, it is identified as a 25th order *AutoRegressive*, i.e. AR(25), model. This is a rather high order model but it is not too unusual in rainfall-flow modelling because of the complex nature of the residuals, which tend to contain evidence of ‘outliers’ (see below). The final residual (one step ahead prediction error) of the model incorporating this AR(25) noise model model, is $\sigma^2 = 0.0031$, which yields a coefficient of determination based on these residuals of $R_T^2 = 0.983$; i.e. the complete stochastic model, explains 98.3% of the flow data. The auto and partial autocorrelation analysis of the final residuals confirms that there is no significant serial correlation left in the series and that they are sensibly white noise, as required. However, they have significantly changing variance (heteroscedastic white noise), varying from zero when there is no flow to the highest variance when the flows are of maximum magnitude. This suggest that some small improvement in the modelling might accrue from an analysis in which this heteroscedasticity is taken into account; e.g. by modelling the heteroscedasticity and incorporating this in the estimation of the model parameters.

Other statistical diagnostic checks on the final model residuals yield reasonable results, given the nature of the data. For instance, the cross correlation function (CCF) between the residuals and the daily temperature series T_t shows that there is no significant correlation and suggests that, in the case of the Canning River data, there is no evidence for any significant temperature related terms in the model,

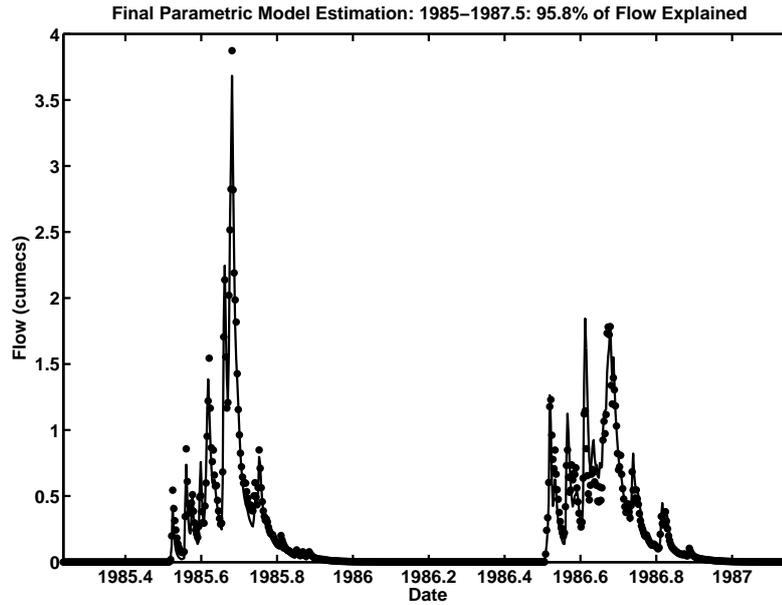


Figure 4. Final parametric estimation of the DBM model: comparison of the simulated DBM model output (full line) and the measured flow data (circular points).

either in the nonlinear effective rainfall function; or as an additive term in the TF model. In other words, it would appear that using the flow as a surrogate measure for catchment storage in the effective rainfall nonlinearity has removed the need to explicitly model any evapo-transpirative effects based on temperature. However, the CCF between the residuals and the input rainfall r_t suggests there is some significant correlation left with the rainfall: this is quite usual with rainfall-flow models and, once more, it probably arises in this case because of the numerous significant outliers. This problem could be obviated by simply assuming that the flow measurements at these outlier samples are missing and replacing them by their expectations. This is quite straightforward when using recursive estimation (see e.g. Young, 1984) but has not been attempted in this case since it might impair the model's ability to characterize the high flow episodes that are important in flood forecasting.

The most valuable evaluation of modelling results is predictive validation on data not used in the identification and estimation analysis. Figure 6 shows two typical predictive validation results obtained by using the estimated DBM model to simulate flow (from the rainfall alone) over other time periods where data are available: the left panel for the period between January 1st 1977 to 13th May 1978; and the right panel between 3rd December 1978 to 15th April, 1980. In the first case, the associated $R_T^2 = 0.954$; while in the latter case, this reduces to $R_T^2 = 0.928$. Clearly, these are very respectable results and they are similar to those obtained from predictive validation analysis carried out over other parts of the data set. They provide confidence in the estimated model, showing that it is robust and

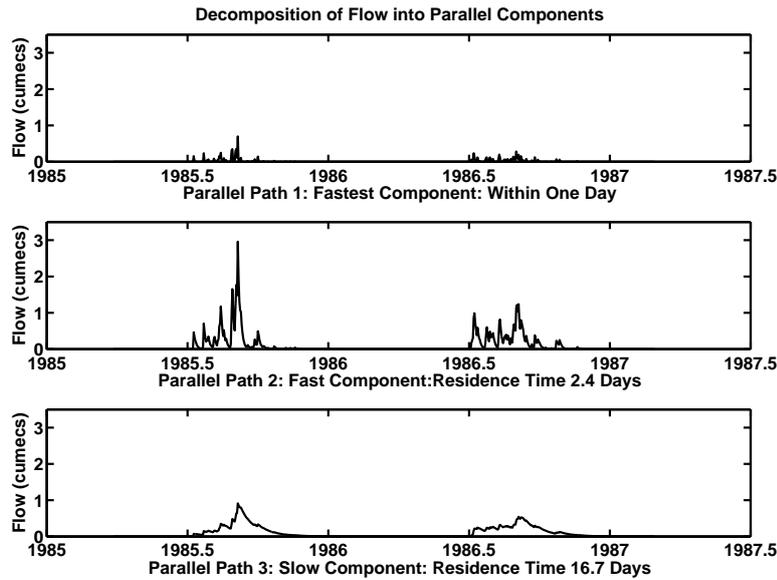


Figure 5. Decomposition of the Canning River flow into parallel instantaneous (top panel), quick (middle panel) and slow (bottom panel) flow components, based on the decomposition of the transfer function in the estimated DBM model.

represents very well the short and long term aspects of the rainfall-flow behaviour in the catchment over all of the available data.

One distinct advantage of the stochastic DBM model is the information it provides on the estimated uncertainties in both the data and the model parameters. Figures 7 and 8 illustrate how this can help in evaluating the model further. Figure 7 shows the estimated uncertainty in the derived residence time parameters for the two main parallel pathways, as revealed by a normalized histogram of the parameters derived from *Monte Carlo Simulation* (MCS) analysis. This analysis involved 2500 stochastic realizations based on the estimated covariance matrix of the model parameters: we see that both residence times are reasonably well defined probabilistically and the distributions are roughly Gaussian in shape, with perhaps a slight skew to larger values in the case of the slow residence time in the left panel.

The same MCS analysis includes an evaluation of the predicted uncertainty associated with the model output. Here, an innovation is the inclusion of a model for the additive heteroscedastic noise (see also the next sub-section). This was obtained by SDP estimation applied to the explanation of the residual variance as a function of the flow y_t and the resulting model was used to generate heteroscedastic noise with these modelled properties in the MCS realizations. We see in Figure 8 that the useful effect of this, in the case of the second predictive validation data set, is to specify the standard error bounds on the DBM model output; bounds that change in width as a function of the flow magnitude and show that the prediction of the peak flow, although in error as regards the measured value (which, of course, may itself be incorrect because high flow measurements are often subject to measurement inaccuracy) lies within the estimated uncertainty bounds.

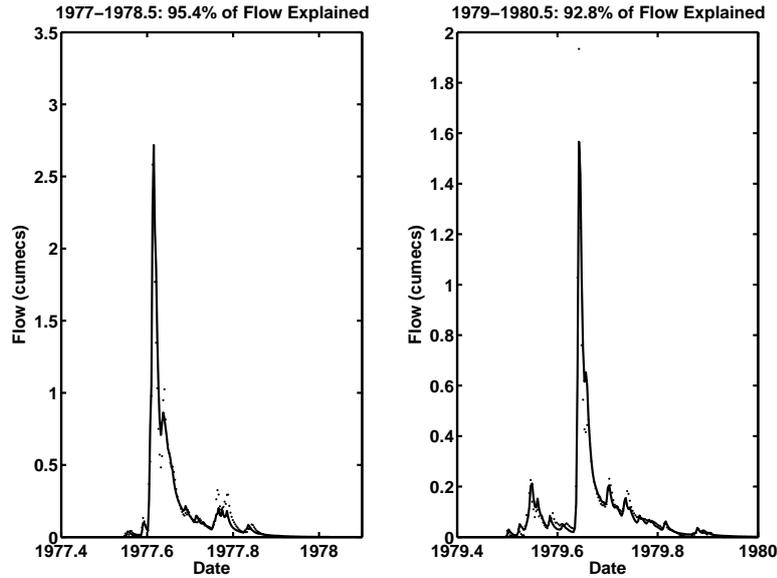


Figure 6. Predictive validation of the DBM model based on two sets of validation data: simulated model output (full line) compared with validation flow data (dots).

(b) Adaptive Forecasting

The main objective of DBM modelling in this example is real-time forecasting. As mentioned in section 7, therefore, the DBM model developed in the previous section, although best at explaining the rainfall-flow data, is not the most suitable model for this application because it requires one-day-ahead forecasting of the rainfall input. A superior alternative is to insert a ‘virtual’ one day delay into the DBM model, as discussed in section 7. The re-estimated parameters of this DBM forecasting model, based on the estimation data set, are as follows:

$$\begin{aligned} \hat{a}_1 &= -1.6243(0.018); & \hat{a}_2 &= 0.6417(0.016); & \hat{\gamma} &= 0.777(0.096); \\ \hat{b}_1 &= 0.209(0.004); & \hat{b}_2 &= -0.191(0.004); \\ \hat{\beta}_1 &= 0.185; & \hat{\alpha}_1 &= 0.679; & \hat{\beta}_2 &= 0.024; & \hat{\alpha}_2 &= 0.946; \end{aligned}$$

and the decomposed form of the model has already been cited in equation (3), with its most useful stochastic state space formulation in equations (4a), (4b) and (4c). This model has an $R_T^2 = 0.949$ very little different to the original DBM model without the time delay. It also passes all the statistical diagnostic tests (except, as before, the CCF between the residuals and the rainfall) and, most importantly, it performs very well in real-time, one-day-ahead forecasting, as shown in Figure 9. This is a plot of the one-day-ahead forecasts for the first validation data set, as obtained from a KF forecasting engine based on an adaptive version of the model.

These forecasts have a coefficient of determination, based on the one-step-ahead forecasting errors of $R_T^2 = 0.918$, which is sort of value one would expect given the DBM modelling results. Parameter adaption is exploited, as described in section 7. The optimized NVR’s, based on minimizing the variance of the one-day-ahead forecasting errors associated with the adaptive parameters are $q_g = 0.0001$ and

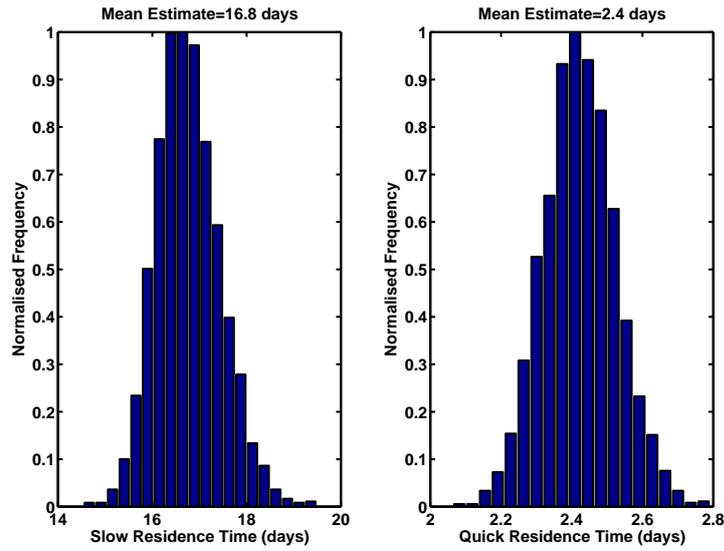


Figure 7. Uncertainty in the derived residence time parameters of the DBM model based on stochastic Monte Carlo analysis: slow residence time (left panel); quick residence time (right panel).

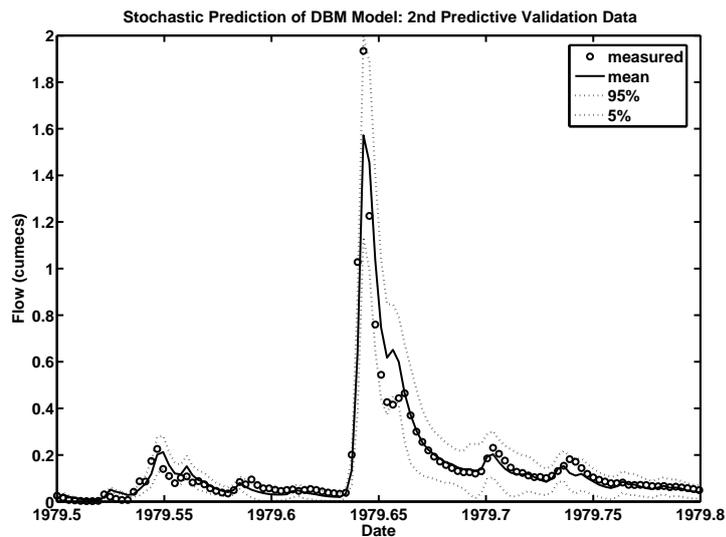


Figure 8. Validation of the DBM model by stochastic Monte Carlo simulation based on the 2nd validation data set: mean model output (full line), 95% confidence bounds (dotted lines) and measured flow (open circular points).

$q_h = 0.169$. The resulting adaptive gain parameter is plotted in Figure 10 and we see that the adaptive adjustment is very small (the non-adaptive value is unity) and makes very little difference in this case. However, this is fortuitous and parameter

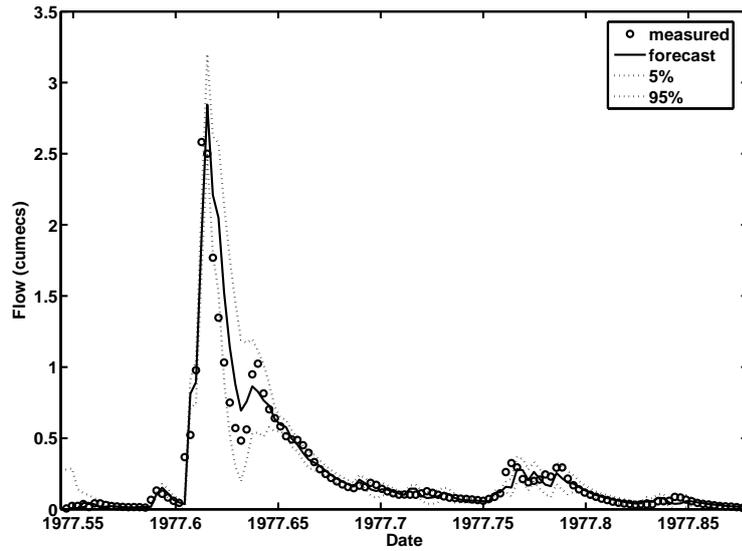


Figure 9. One-day-ahead forecasts for the 1st validation data set produced by the adaptive Kalman Filter forecasting system based on the DBM forecasting model: forecast (full line), 95% confidence bounds (dotted line) and flow data (open circular points).

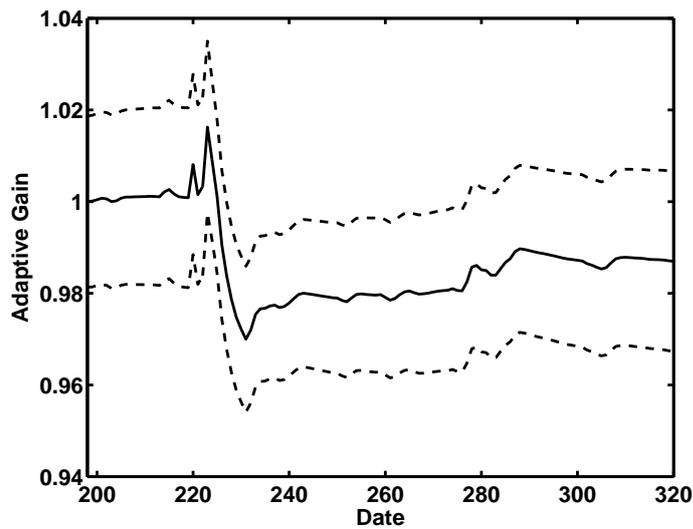


Figure 10. Adaptive gain estimate associated with the forecasting results in Figure 9: recursive estimate of gain parameter (full line) and estimated standard error bounds (dashed lines).

adaptation should always be implemented to handle unforeseen circumstances. On the other hand, the adaptive variance parameter makes considerable difference to

the results, a fact that is most visible in the changing width of the 95% confidence band in Figure 9.

Finally, note that the state space model used to obtain the above forecasting results does not contain any states associated with the identified additive noise ξ_t which, it will be recalled, is identified as an AR(25) stochastic process. This noise model could have been embedded in the state space model and it should improve the forecasts to some extent. However, since it would also considerably enlarge the state dimension (adding 25 more stochastic state variables) and so add complexity to the forecasting system, it was not included in the present illustrative analysis.

9. Conclusions

This Chapter describes some recent advances in stochastic modelling and forecasting that provide the basis for the implementation of real-time flow and flood forecasting/warning systems. It argues that deterministic reductionist (or ‘bottom-up’) simulation models are inappropriate for real-time forecasting because of the inherent uncertainty that characterizes river catchment dynamics and the problems of model over-parametrization that are a natural consequence of the reductionist philosophy. The advantages of alternative Data-Based Mechanistic (DBM) models, statistically identified, estimated and validated in an inductive manner directly from rainfall-flow data, are discussed. In particular, the Chapter shows how nonlinear, stochastic, transfer function models can be developed using powerful methods of recursive time series analysis. Not only are these models able to characterize well the rainfall-flow dynamics of the catchment in a parametrically efficient manner but, by virtue of the DBM modelling strategy, they can also be interpreted in hydrologically meaningful terms. Most importantly in the forecasting context, the models are also in an ideal form for incorporation into a data assimilation and forecasting engine based on a special, adaptive version of the Kalman Filter algorithm.

The practical example described in the paper demonstrates how, with sufficient rainfall-flow data and no available rainfall forecasts, the approach proposed here can generate useful flow forecasts for one day ahead; forecasts that could form the basis for flood warning system design. Such a system would be a natural development of the Dumfries flood warning system (Lees *et al.*, 1994), which was designed from a similar DBM modelling standpoint and has been operating successfully without major modification since 1991. The methodological advances described in the present paper would ensure much improved performance from such a system but the basic minimalist design and low economic cost of development would be retained. Both of these recursive approaches to real-time forecasting can be contrasted with more conventional, non-recursive, real-time forecasting procedures proposed previously. A typical example is the adaptive scheme suggested by Brath & Rosso (1993) which addresses some of the same statistical issues raised in the present paper. However, it operates on an event basis rather than continuously; it uses repeated *en-bloc* optimization rather than recursive estimation; it is based on a simple conceptual model with *a priori* assumed structure and parameterization; and it is computationally much more demanding.

Of course, there remain a number of methodological problems still to be solved. The DBM models discussed in the paper perform well but they cannot be considered completely satisfactory while the model residuals retain some mildly unsatis-

factory statistical characteristics. In particular, the correlation remaining between the residuals and the rainfall input shows that the model is still not fully explaining the complete rainfall-flow process (although the remaining unexplained variance represents only a small proportion of the total variance). This limitation of the current DBM models (shared, the author believes, by *all* current rainfall-flow models, whatever their type) is almost certainly due to deficiencies in the effective rainfall nonlinearity and possibly the presence of other, smaller nonlinearities in the system as yet unquantified. There is clear need for more research on this fascinating subject and, although such research would require the analysis of a large and comprehensive rainfall-flow data base covering a wide array of different catchment behaviour, it would provide useful information for *all* existing rainfall-flow modelling studies, not just those discussed in this paper.

Finally, it is interesting to compare the DBM approach to hydrological modelling described in this Chapter with the alternative numerical Bayesian approach, as exemplified in a number of recent papers (e.g. Thiemann *et al.*, 2001: see also the associated comment by Beven and Young, 2003). While it is clear that the latter methodology has great generality and shows great promise, it is computationally very intensive when compared with the DBM methodology: while the DBM computation takes but a few seconds on a typical desktop computer, the Bayesian

Appendix A. Transfer Function (TF) Models

† Most applications of TF models in hydrology tend to use discrete-time TF models because most of the literature on the identification and estimation of TF models concentrates on this form. However, as we shall see, the continuous-time TF model is more transparent and immediately interpretable in a physically meaningful manner. In order to introduce TF models, therefore, let us consider first a conceptual catchment storage equation in the form of a continuous-time, linear storage (‘bucket’, ‘tank’ or ‘reservoir’) model: see, for example, the review papers by O’Donnell, Dooge and Young in Kraijenhoff and Moll (1986) or more comprehensive treatments, such as the recent books by Beven (2001) and Dooge and O’Kane (2003). Here, the rate of change of storage in the channel is defined in terms of water volume entering the linear storage element (e.g. river reach) in unit time, minus the volume leaving in the same time interval, i.e.,

$$\frac{dS(t)}{dt} = GQ_i(t - \tau) - Q_o(t) \quad (\text{A } 1)$$

where $Q_i(t - \tau)$ represents the input flow rate delayed by a pure time or ‘transport’ delay of τ time units to allow for pure advection; and G is a gain parameter inserted to represent gain (or loss) in the system. Making the reasonable and fairly common assumption that the outflow is proportional to the storage at any time, i.e.,

$$Q_o(t) = \rho S(t)$$

and substituting into (A 1), we obtain,

$$T \frac{dQ_o(t)}{dt} = GQ_i(t - \tau) - Q_o(t); \quad (\text{A } 2)$$

† This Appendix is taken in part from Young (2005) which also describes the historical context of TF models and their use in hydrology.

This equation is a first order, linear differential equation model whose response, from an initial steady flow condition, to a unit impulsive change Q_i^{imp} of the input flow at time $t = t_0$, is given by

$$Q_o(t) = Q_e + Q_i^{imp} e^{-(t-t_0)/T},$$

where Q_e is the initial steady flow level. This has a typical hydrograph recession shape, with a decay *Time Constant*, T , that defines the *Residence Time* of the model. As we shall see later, combinations of two or more such first order models, exhibit a typical unit hydrograph form (e.g. Dooge, 1959; Beven, 2001).

By introducing the derivative operator s , i.e. $s = d/dt$, and collecting like-terms together, it is easy to see that equation (A 2) can be written as,

$$(1 + Ts)Q_o(t) = GQ_i(t - \tau)$$

so that, dividing throughout by $1 + Ts$, we obtain the following continuous-time TF form of equation (A 2),

$$Q_o(t) = \frac{G}{1 + Ts} Q_i(t - \tau) \quad (\text{A } 3)$$

where,

$$H(s) = \frac{G}{1 + Ts}$$

represents the TF in terms of the derivative operator s^\dagger .

(a) *Physically Interpretable Parameters*

The TF model (A 3) is characterized by three parameters: G , T and τ . However, there are five, physically interpretable model parameters associated with the model that are worth discussing. The *Steady State Gain* (SSG), denoted by G , is obtained by setting the s operator in the TF to zero (i.e. $d/dt = 0$ in a steady state). It shows the relationship between the equilibrium output and input values when a steady input is applied. For this reason, *if the input and output have similar units*, G is ideal for indicating the physical losses or gains occurring in the system. In the case of a flow-routing model, for example, it indicates whether water has been added ($G > 1$) or lost ($G < 1$) between the upstream and downstream boundaries; and the percentage of water lost or gained can be defined by *Loss Efficiency* $LE = 100(1 - G)$, which will be negative if $G > 1.0$. As pointed out above, the *Residence Time* or *Time Constant* T is the time required for the storage element output to decay to e^{-1} or 0.3679 of its maximum value in response to an impulsive input. Finally the pure *Advective Time Delay* τ indicates the time it takes for a flow increase upstream to be first detected downstream: and $T_t = T + \tau$ defines the *Travel Time* of the system. These five parameters typify the equilibrium and dynamic characteristics of the TF model and provide a physical interpretation of the TF model in terms of its mass transfer and dispersive characteristics.

† The same letter s (or sometimes p) is used to represent the related Laplace transform operator. Considered in these Laplace transform terms, it is possible to utilize Laplace transform methods to handle initial conditions on the variables in the model and analytically compute its response (here $Q_o(t)$) to variations in input variable (here $Q_i(t)$). However, this is not essential in the present context, although interested reader should find the Appendix is a good primer for the study of Laplace transform methods (e.g. Schwarzenbach and Gill, 1975).

(b) *TF Manipulation and Block Diagrams*

The first order TF model (A 3) is often written in the form,

$$Q_o(t) = \frac{g_0}{s + f_1} Q_i(t - \tau) \quad \text{i.e.} \quad H(s) = \frac{g_0}{s + f_1} \quad (\text{A } 4)$$

where,

$$g_0 = \frac{G}{T}; \quad f_1 = \frac{1}{T}$$

because this is the form in which the model is normally estimated (see later). Typically, a Channel or Flow Routing model for a river catchment will contain a number of elemental models, such as (A 3) or (A 4), connected in a manner that relates to the structure of the catchment. For instance, a serial connection of n such elements constitutes the lag-and-route model of a single river channel (Meijer, 1941; Dooge, 1986) and, with $\tau = 0$ and all elements identical, it becomes the well known ‘Nash Cascade’ model (Nash, 1959). More complex river systems can be represented by a main channel of this type, with tributaries modelled in a similar manner. Also, a typical TF model between effective rainfall and flow often contains a parallel connection of two or more such storage elements (see Young, 1992, 2001a,b, 2002a,b, 2003).

The TF formulation allows for the visual representation of a total system model in the form of a ‘*Systems Block Diagram*’. Figure 11 is a typical example of such diagram that represents a catchment model consisting of a effective rainfall-flow sub-model involving two *different* first order TFs of the form (A 3) with $\tau = 0$,

$$H_1(s) = \frac{G_1}{1 + T_1 s}; \quad H_2(s) = \frac{G_2}{1 + T_2 s},$$

that are connected in parallel. The flow output of this sub-model forms the input to a flow-routing sub-model composed of two *identical*, first order TFs,

$$H_3(s) = \frac{G_3}{1 + T_3 s},$$

again of the form (A 3) with $\tau = 0$, but this time connected in series as a ‘Nash Cascade’. The upstream input to this complete system is an effective rainfall measure (see main text). The downstream output of the complete system is denoted by $y(t)$.

One advantage of the TF formulation of the model shown in Figure 11 is that it allows for the computation, using ‘*Block Diagram Algebra*’, of a single, multi-order TF that represents the total system. Here, TFs connected in parallel are additive; while those connected in series are multiplicative. Consequently, in this case, the two sub-models can be represented by the following composite TFs:

$$H_{r,f}(s) = H_1(s) + H_2(s); \quad H_{f,f}(s) = H_3(s).H_3(s)$$

So that, with the above definitions of $H_1(s)$, $H_2(s)$ and $H_3(s)$,

$$H_{r,f}(s) = \frac{G_1}{1 + T_1 s} + \frac{G_2}{1 + T_2 s} = \frac{(G_1 + G_2) + (G_1 T_2 + G_2 T_1) s}{(1 + T_1 s)(1 + T_2 s)}$$

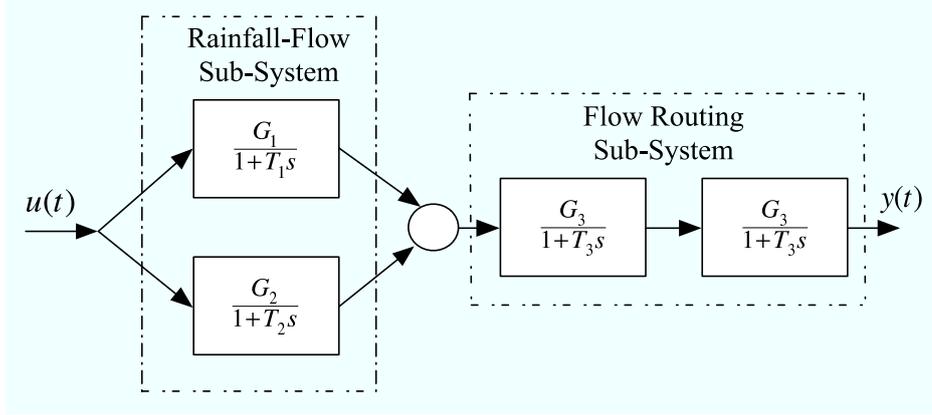


Figure 11. Block diagram of a hypothetical catchment model consisting of a parallel pathway, rainfall-flow sub-system in series with a two reach, flow-routing sub-system.

and

$$H_{ff}(s) = \frac{(G_3)^2}{(1 + T_3s)(1 + T_3s)}$$

Now, since $H_{rf}(s)$ and $H_{ff}(s)$ are connected in series, the TF of the total system $H(s)$ is obtained as the multiplication of these two composite TFs: i.e.,

$$H(s) = H_{rf}(s) \cdot H_{ff}(s) = \frac{(G_1 + G_2) + (G_1T_2 + G_2T_1)s}{(1 + T_1s)(1 + T_2s)} \cdot \frac{(G_3)^2}{(1 + T_3s)(1 + T_3s)}$$

Multiplying out these expressions, we see that the complete $H(s)$ is a 4th order TF that can be manipulated to the form:

$$H(s) = \frac{g_0 + g_1s}{s^4 + f_1s^3 + f_2s^2 + f_3s + f_4} \quad (\text{A } 5)$$

where we will leave the definition of the parameters $f_i, i = 1, 2, \dots, 4$ and $g_j, j = 0, 1$ in (A 5), as an exercise for the reader (*hint*: carry the above analysis with the TF representation (A 4) rather than (A 3): then it will be clear why the former representation is better for analysis, although it lacks the direct physical interpretation of the latter, which provides a better form for the block diagram representation).

(c) The General, Multi-Order TF Model

It is clear from the above example that, in general, serial, parallel (or even feedback[†]) connections of elemental first order TF models, such as (A 3) or (A 4), lead to a multi-order TF model that takes the general TF form:

$$x(t) = \frac{G(s)}{F(s)}u(t - \tau) \quad y(t) = x(t) + \xi(t) \quad (\text{A } 6)$$

[†] The block diagram algebra for a feed back connection is a little more complicated but, since it is not particularly relevant in the current context, the interested reader should consult a standard text on the subject (e.g. Schwarzenbach and Gill, 1975) to find the details.

where $F(s)$ and $G(s)$ are polynomials in s of the following form:

$$\begin{aligned} F(s) &= s^p + f_1 s^{p-1} + f_2 s^{p-2} + \dots + f_p \\ G(s) &= g_0 s^q + g_1 s^{q-1} + g_2 s^{q-2} + \dots + g_q \end{aligned}$$

in which p and q can take on any positive integer values. Here $u(t)$ and $x(t)$ denote the deterministic input and output signals of the system at its upstream and downstream boundaries, respectively; τ is a pure advective time (transport) delay affecting the input signal $u(t)$; and $y(t)$ is the observed output, which is assumed to be contaminated by a noise or stochastic disturbance signal $\xi(t)$. This noise is assumed to be independent of the input signal and it represents the aggregate effect, at the down stream boundary, of all the stochastic inputs to the system, including distributed unmeasured inputs, measurement errors and modelling error. Multiplying throughout equation (A 6) by $F(s)$ and converting the resultant equation to alternative ordinary differential equation form, we obtain:

$$\frac{d^p y(t)}{dt^p} + f_1 \frac{d^{p-1} y(t)}{dt^{p-1}} + \dots + f_p y(t) = g_0 \frac{d^q u(t-\tau)}{dt^q} + \dots + g_q u(t-\tau) + \eta(t) \quad (\text{A } 7)$$

where $\eta(t) = F(s)\xi(t)$ is a modified noise signal generated by the manipulation of the equation. Depending on the objectives of the modelling study, it may be necessary, in a complete system consisting of many sub-elements such as (A 3) or (A 4), to consider noise inputs *within* the system, associated with collections of sub-elements that have distinct physical meaning: e.g. stochastic lateral inflows. The structure of this model, in either form (A 6) or (A 7), is defined by the triad $[p \ q \ \tau]$.

(d) Discrete-Time, Sampled Data TF Models

To date, the most popular form of TF modelling has been carried using the discrete-time (DT) equivalents of the models (A 4) and (A 6). In the case of equation (A 4), this discrete-time TF model takes the form:

$$Q_{o,t} = \frac{b_0}{1 + a_1 z^{-1}} Q_{i,t-\delta} \quad (\text{A } 8)$$

Here, $Q_{o,t}$ is the downstream flow measured at the t^{th} sampling instant, that is at time $t\Delta t$, where Δt is the sampling interval in time units. $Q_{i,t-\delta}$ is the input flow at time $(t-\delta)\Delta t$ time units previously, where δ is the advective time delay, normally defined as the nearest integer value of $\tau/\Delta t$ (thus incurring a possible approximation error); and z^{-1} is the backward shift operator, i.e. $z^{-r} Q_{o,t} = Q_{o,t-r}$. Of course, this model can be written in its ‘difference equation’ form, namely:

$$Q_{o,t} = -a_1 Q_{o,t-1} + b_0 Q_{i,t-\delta} \quad (\text{A } 9)$$

which is obtained by simple cross multiplication and application of the z^{-1} operator. This reveals that the flow $Q_{o,t}$ at the t^{th} sampling instant is a proportion $-a_1$ (note that, in the present context, a_1 will be a negative number less than unity, so that this is a positive proportion) of its value $Q_{o,t-1}$ at the previous $(t-1)^{\text{th}}$ sampling instant, plus a proportion b_0 of the delayed upstream flow input $Q_{i,t-\delta}$ measured δ sampling instants previously.

The values of the parameters a_1 and b_0 in equations (A 8) and (A 8) can be related to the parameters of the model (A 4) in various ways depending upon how the input flow $Q_i(t)$ is assumed to change over the sampling interval between the measurement of $Q_{i,t-1}$ and $Q_{i,t}$ (since it is not measured over this interval). The simplest and most common assumption is that it remains constant over this interval (the so-called zero-order hold, ZOH, assumption), in which case the relationships are as follows:

$$a_1 = -\exp(-f_1 \Delta t) \quad b_1 = \frac{g_0}{f_1} \{1 - \exp(-f_1 \Delta t)\} \quad (\text{A } 10)$$

Note that, because these relationships are functions of the sampling interval Δt , for every unique CT model such as (A 4), there are infinitely many DT equivalents (A 8), depending on the choice of Δt , all with different parameter values defined in (A 10). Following from the definition of this first order DT model at the chosen Δt , the general multi-order equivalent of the general CT model (A 6) is defined as follows:

$$x_t = \frac{B(z^{-1})}{A(z^{-1})} u_{t-\delta} \quad y_t = x_t + \xi_t \quad (\text{A } 11)$$

where,

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n} \\ B(z^{-1}) &= b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_m z^{-m} \end{aligned}$$

Normally $n = p$ but m may be equal or greater than q . The structure of this DT model is defined by the triad $[n \ m \ \delta]$. Note finally that the TFs in both (A 11) and (A 6) are composed of ratios of polynomials (in z^{-1} and s respectively), so they are sometimes referred to as *Rational Transfer Functions*.

(e) *The Unit Hydrograph and Finite Impulse Response TF Models*

The reader can verify that the division of the numerator polynomial $B(z^{-1})$ by the denominator polynomial $A(z^{-1})$ in the TF model (A 11) normally results in a infinite dimensional polynomial $G(z^{-1}) = g_0 + g_1 z^{-1} + g_2 z^{-2} + \dots + g_\infty z^{-\infty}$, so that the equation can also be written in the alternative form:

$$y_t = \sum_{i=\delta}^{\infty} g_i u_{t-i} + \xi_t; \quad g_j, j = 0, 1, \dots, \delta - 1 = 0 \quad (\text{A } 12)$$

This will be recognized as the discrete-time form of the convolution integral equation associated with the solution of differential equations (here with a pure time delay of δ sampling intervals or $\delta \Delta t$ time units) and, once again, we see that the TF model is simply a discrete-time equivalent of a continuous-time differential equation model.

Equation (A 12) has important connotations in hydrology because, if the noise $\xi_t = 0$ for all t , then its unit impulse response, i.e. the response of the equation to a unit impulse input: $u_t = 1.0$ for $t = 1$; $u_t = 0$ for all $t > 0$), is equivalent to the hydrological unit hydrograph. Sometimes, indeed, the equation (A 12) is referred to as a TF model, although its infinite dimensional nature is an obvious restriction. Despite this disadvantage, some hydrologists have used the equation directly in RF modelling by considering it in a *Finite Impulse Response* (FIR) form, in which the

upper limit of the summation is set to some value $r < \infty$ where it is considered that the ordinates of the impulse response have become small enough to be ignored.

Unfortunately, it can be shown that the number of FIR model parameters (or 'weights'), g_i , $i = 1, 2, \dots, r$, is nearly always much larger than the number of parameters in the rational TF form (A 11). As a result, the statistical estimates of these parameters will normally have unacceptably high variance and have to be constrained in some manner. For instance, Natale and Todini (1976) use quadratic programming to compute the constrained least squares estimates of the FIR parameters in order to ensure that they are all positive and, if necessary for conservation purposes, they all sum to unity (i.e. the model has unity steady state gain). Even with this approach, however, it is difficult to recommend the FIR model because the rational TF model (A 11), normally with far fewer parameters, is not only entirely equivalent to the infinite dimensional IR model, without any approximation, it is also easier to estimate from rainfall-flow data using the methodology discussed in this Chapter.

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